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EFFECT OF WALL RADIATION ON THERMAL INSTABILITY IN A VERTICAL CYLINDER

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Abstract—Wall heat radiation in a long vertical cylinder heated from below increases critical Rayleigh number by up to a factor of three. The analysis presented explains why initiation of natural convection in a gas-filled honeycomb structure occurs at a Rayleigh number higher than that necessary when the same honeycomb is filled with an infrared-opaque liquid.

NOMENCLATURE

A, area;
b, vertical wave number;
d, diameter;
F, shape factor;
H, irradiation;
H^*, dimensionless irradiation;
K, fluid thermal conductivity;
K_e, equivalent infinite wall conductivity;
K_e^*, equivalent conductivity with radiation;
K_w, wall material conductivity;
n, circumferential wave number;
q_r, radiant heat flux into wall;
Q, dimensionless amplitude of radiation;
r, radius;
R, Rayleigh number;
$S_n(b)$, radiation integral;
t_w, wall thickness;
T_0, absolute temperature level;
T', temperature perturbation;
T'_w, wall temperature perturbation;
z, vertical coordinate;
β, vertical temperature gradient;
ε, wall emissivity;
ζ, vertical variable of integration;
θ, radial dependence of T';
ξ, circumferential variable;
σ, Stefan–Boltzmann constant;
ϕ, circumferential angle.

INTRODUCTION

FROM the works of Ostroumov [1], Yih [2] and others [3, 4] it is well-known that the Rayleigh number at which thermal convection initiates in a long vertical cylinder filled with fluid and heated from below varies significantly with wall conductivity and wall thickness. Ostroumov [1] presents a derivation for the conductivity K_e of an equivalent infinitely thick wall in terms of the actual wall conductivity and thickness. For a cylinder wall insulated on the exterior, his result, corrected for a typographical error, is

$$K_e = K_w[1 - d^2/(d + 2t_w)^2]/[1 + d^2/(d + 2t_w)^2]. \qquad (1)$$

For a thin wall equation (1) reduces to

$$K_e = 2K_w t_w/d. \qquad (2)$$

Ostroumov then gives critical Rayleigh number R based on the mean vertical temperature gradient and radius for a cylinder filled with a fluid of conductivity K in his Table II as a function of K_e/K. Rayleigh number increases by a factor of 3 as the conductivity ratio increases from zero to infinity.

Applied workers interested in the prediction of heat transfer through gas filled nonmetallic honeycomb structures may, in applying the results of Ostroumov, be led to a value of critical Rayleigh number up to three times too small, if they fail to account for thermal radiation exchange between wall elements. It is the purpose of this work to derive the wall radiation terms affecting stability in a long vertical right circular cylinder filled with a diathermanous gas and subjected to a destabilizing axial temperature gradient.

THEORY

A fluid with an undisturbed temperature $T_0 - \beta z$ may be perturbed so that the temperature changes a small amount T'. Ostroumov [1] and Yih [2] considered a disturbance of the form

$$T'(r, \phi, z) = (\beta d/2)\, \theta(r) \cos bz \cos n\phi, \quad (3)$$

where, because of the principle of exchange of stabilities [5], time dependency does not enter explicitly. The perturbation is subject to side wall boundary conditions.

$$-K_e \frac{\partial T'_w}{\partial r}\bigg|_{r = d/2} = -K \frac{\partial T'}{\partial r}\bigg|_{r = d/2}, \qquad T'_w|_{r = d/2} = T'|_{r = d/2}. \quad (4)$$

Radiation in a diathermanous fluid affects only the boundary condition; it may be accounted for by adding the radiant heat flux to equation (4)

$$-K_e \frac{\partial T'_w}{\partial r}\bigg|_{r = d/2} = -K \frac{\partial T'}{\partial r}\bigg|_{r = d/2} + q_r, \qquad T'_w|_{r = d/2} = T'|_{r = d/2}. \quad (4a)$$

It may be seen by comparison of equations (3), (4) and (4a) that Ostroumov's analysis can be taken over in its entirety, provided q_r is of the form

$$q_r = K\beta Q \cos bz \cos n\phi, \quad (5)$$

where Q is a dimensionless constant. This situation will be shown to hold true for a gray diffuse interior cylinder wall and small temperature differences. The result will be that a radiation contribution should be added to K_e to give K_e^* to be used in place of K_e in Ostroumov's results.

Consider a location ϕ_1, z_1 on the inside wall of a long cylinder of unit radius. The irradiation is for gray, perfectly diffuse walls [6]

$$H(\phi_1, z_1) = \int_{-\infty}^{+\infty} \int_{-\pi}^{+\pi} [\varepsilon\sigma T_2{}^4 + (1 - \varepsilon)\, H(\phi_2, z_2)] \frac{dF}{dA}\, d\phi_2\, dz_2, \quad (6)$$

$$\frac{dF}{dA} = \frac{[1 - \cos(\phi_2 - \phi_1]^2}{\pi\{2[1 - \cos(\phi_2 - \phi_1)] + (z_2 - z_1)^2\}^2}. \quad (7)$$

For small $\beta d/T_0$ it is permissible to linearize,

$$\sigma T_2^4 = \sigma T_0^4 + 4\sigma T_0^3(-\beta z_2 + T'_w), \quad (8)$$

$$H(\phi, z) = \sigma T_0^4 - 4\sigma T_0^3 \beta z + 4\sigma T_0^3 (\beta d/2)\, H^*(\phi, z). \quad (9)$$

Introducing equations (8) and (9) into (6), changing variables of integration from z_2 to $\zeta = z_2 - z_1$, and introducing θ from equation (3) and (4) gives

$$H^*(\phi_1, z_1) = \int_{-\infty}^{+\infty} \int_{-\pi}^{+\pi} [\varepsilon\theta(1) \cos b(\zeta + z_1) \cos n\phi_2 + (1 - \varepsilon)\, H^*] \frac{dF}{dA}\, d\phi_2\, d\zeta. \quad (10)$$

The term arising from $4\sigma T_0^3 \beta z_2$ drops out by virtue of the symmetry of the kernel dF/dA. Multiplying and dividing under the integral by $\cos n\phi_2 \cos bz_2$ and changing the variable of integration from ϕ_2 to $\xi = \phi_2 - \phi_1$ yields

$$H^*(\phi_1, z_1) = \int_{-\infty}^{+\infty} \int_{-\pi}^{+\pi} \left[\varepsilon\theta(1) + (1-\varepsilon) \frac{H^*(\phi_2, z_2)}{\cos n\phi_2 \cos bz_2} \right]$$

$$[\cos b\zeta \cos bz_1 - \sin b\zeta \sin bz_1]$$
$$[\cos n\xi \cos n\phi_1 - \sin n\xi \sin n\phi_1]$$

$$\frac{dF}{dA} d\xi \, d\zeta. \quad (11)$$

Since the kernel dF/dA is even with respect to ξ and ζ, the solution is

$$H^*(\phi_1, z_1) = \{\varepsilon S_n(b)/[1 - (1-\varepsilon) S_n(b)]\} \theta(1) \cos n\phi_1 \cos bz_1, \quad (12)$$

$$S_n(b) = \int_{-\infty}^{+\infty} \int_{-\pi}^{+\pi} \frac{[1 - \cos \xi]^2 \cos n\xi \cos b\zeta}{\pi[2(1 - \cos \xi) + \zeta^2]^2} d\xi \, d\zeta. \quad (13)$$

For either the adiabatic or the perfectly conducting wall the mode of disturbance $b = 0$, $n = 1$ is the most unstable one. These values are consistent with the Ostroumov analysis. Equation (13) yields in this case $S_1(0) = -\frac{1}{3}$.

The heat flux into the wall is given by the radiation absorbed less that emitted, $\varepsilon H - \varepsilon\sigma T_w^4$, which gives

$$q_r = -4\varepsilon\sigma T_0^3 (\beta d/2) \{[1 - S_n(b)]/[1 - (1-\varepsilon)S_n(b)]\} \theta(1) \cos bz \cos n\phi. \quad (14)$$

Comparison of equations (5) and (14) yields the dimensionless quantity Q which may be substituted into equation (4a) to obtain the equivalent conductivity including the effect of radiation. For $n = 1$, $b = 0$, there is found

$$K_e^* = K_e + (\sigma T_0^3 d)[2\varepsilon/(1 - \varepsilon/4)]. \quad (15)$$

DISCUSSION

Consider a commercially available phenolic fiberglass honeycomb, Hexcell Products HRP 3/4 GF 13–1.9, with an inscribed circle diameter $d = 19{\cdot}1$ mm and half wall thickness $t_w = 0{\cdot}102$ mm. At room temperature $K_e/K_{air} = 0{\cdot}14$, while K_e^*/K_{air} is approximately 1·0. From Ostroumov's Table II the first value yields $R = 75$, while the second value yields the correct result of 105, a value 40 per cent larger. The difference can be larger for larger diameters; higher temperatures; thinner, less conducting walls; or less conducting gases.

The effect of wall radiation to stabilize an infrared transparent fluid in a cylinder with nonconducting walls, demonstrated by our development leading to equation (15), is readily understandable from a physical point of view. If a perturbation starts an upflow in one half of a cylinder and a downflow in the other, the wall close to the warm upflow drains energy from it and radiates the energy to the wall close to the cool downflow and in this manner helps damp the convection.

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EFFET DU RAYONNEMENT DE LA PAROI SUR L'INSTABILITÉ THERMIQUE DANS UN CYLINDRE VERTICAL

Résumé—Le rayonnement thermique de la paroi dans un long cylindre vertical chauffé depuis sa base accroît le nombre de Rayleigh critique en le multipliant par un facteur atteignant trois. L'analyse présentée explique pourquoi l'initiation de la convection naturelle dans une structure en nid d'abeille remplie de gaz se produit à un nombre de Rayleigh plus élevé que dans le cas de la même structure emplie d'un liquide opaque au rayonnement infrarouge.

DIE WIRKUNG DER WAND-STRAHLUNG AUF DIE THERMISCHE INSTABILITÄT IN EINEM VERTIKALEN ZYLINDER

Zusammenfassung—Wärmestrahlung von der Wand in einem langen vertikalen Zylinder, der von unten beheizt wird, vergrössert die kritische Rayleigh-Zal bis zu einem Faktor von drei. Die gegebene Analyse erklärt, warum der Beginn von natürlicher Konvektion in einer gas-gefüllten Honigwabenstruktur bei einer höheren Rayleigh-Zahl geschieht, als dafür nötig ist, falls die gleiche Honigwabe mit einer im Infraroten undurchsichtigen Flüssigkeit gefüllt ist.

ВЛИЯНИЕ ИЗЛУЧЕНИЯ ОТ СТЕНОК НА ТЕРМИЧЕСКУЮ НЕУСТОЙЧИВОСТЬ В ВЕРТИКАЛЬНОМ ЦИЛИНДРЕ

Аннотация—Излучение тепла от стенок длинного вертикального цилиндра обогреваемого снизу повышает критическое число Райлей по крайней мере в три раза. В данном анализе объясняется почему возникновение естественной конвкции в наполненных газом медовых сотах происходит при более высоком числе Райлей чем это требуется, когда те же соты наполнены инфракрасной непрозрачной жидкостью.